BEVERIDGEAN PHILLIPS CURVE

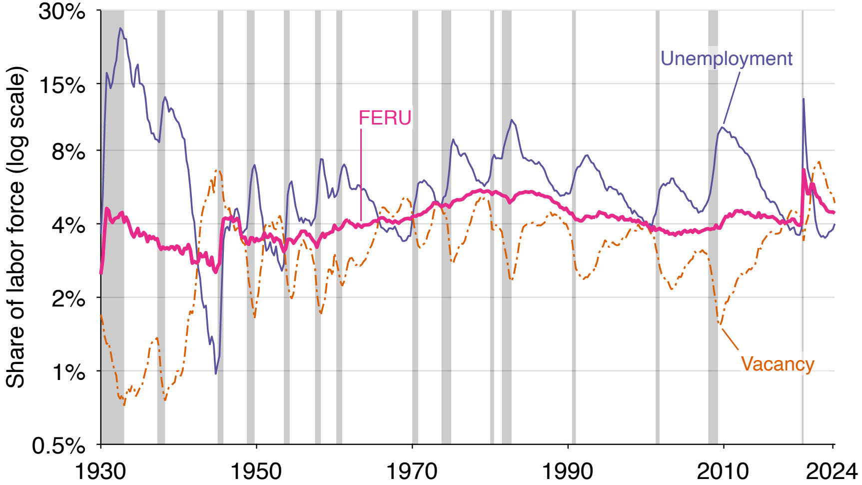
Pascal Michaillat, Emmanuel Saez

May 2025

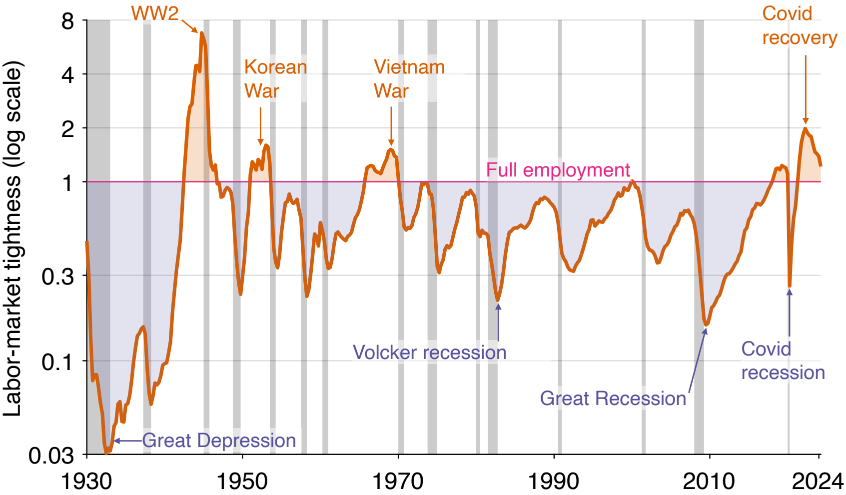
FEDERAL RESERVE'S DUAL MANDATE

- Federal Reserve Reform Act of 1977: "To promote effectively the goals of maximum employment, stable prices"
- Statement on Longer-Run Goals & Monetary Policy Strategy:
  - Stable prices: $\pi^* = 2\%$
  - Maximum/full employment: $u^* = ??$
- Michaillat & Saez (2024): $u^* = \sqrt{u \cdot v}$
  - u^* ensure a socially efficient allocation of labor
  - u^* = full-employment rate of unemployment = FERU
  - FERU $\neq$ NAIRU

FERU AVERAGES 4.1% OVER 1930–2024



US ECONOMY IS GENERALLY NOT AT FULL EMPLOYMENT



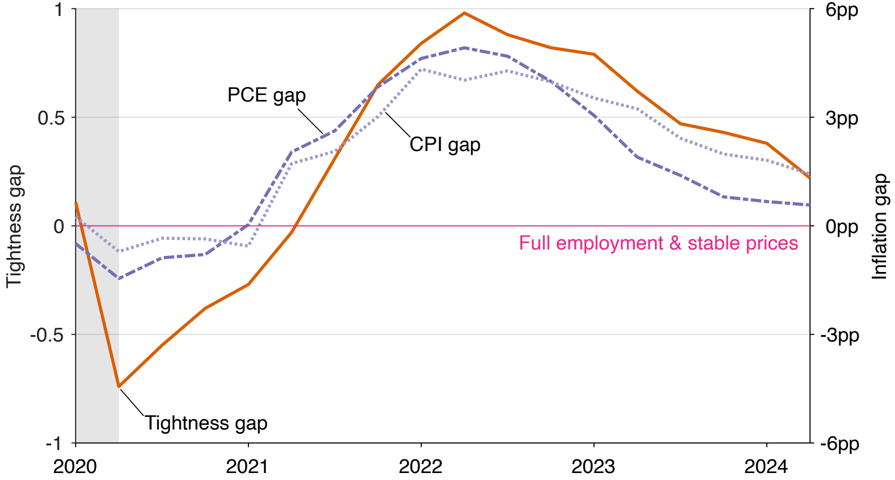
DO THE PRICE-STABILITY AND FULL-EMPLOYMENT MANDATES OVERLAP?

TWO MANDATES ARE USUALLY THOUGHT TO BE INCONSISTENT

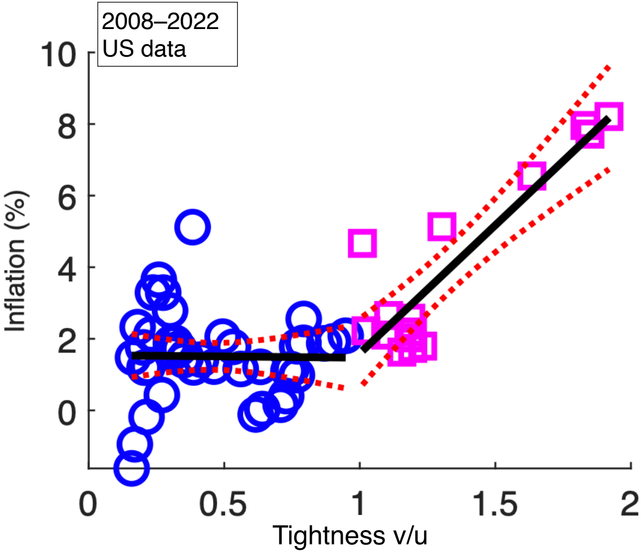
- Traditional Phillips curve: no guarantee that (u^*, π^*) is on curve
- Accelerationist Phillips curve: no guarantee that NAIRU provides an efficient allocation of labor
- New Keynesian Phillips curve with unemployment: wage rigidity breaks down divine coincidence (Blanchard & Gali 2010)

$\rightsquigarrow$ "Divine" coincidence is regarded as unrealistic (Blanchard & Gali 2007)

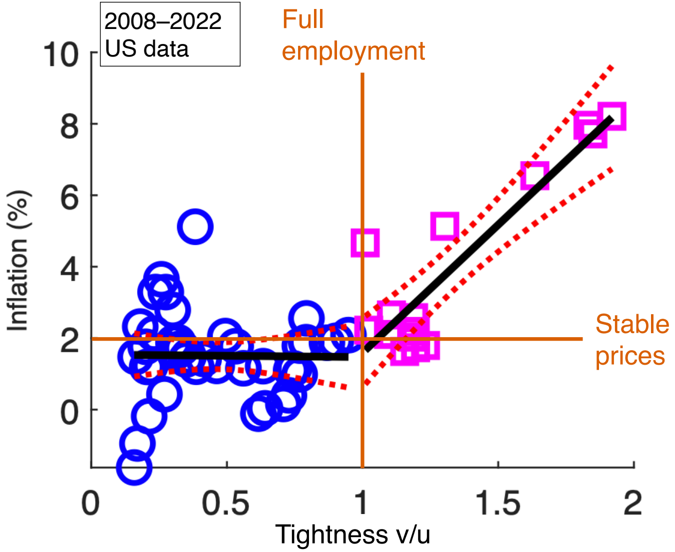
BUT, DIVINE COINCIDENCE APPEARS IN RECENT DATA



ALSO AT LONGER HORIZON (BENIGNO & EGGERTSSON 2023)



ALSO AT LONGER HORIZON (BENIGNO & EGGERTSSON 2023)



DIVINE COINCIDENCE ALSO APPEARS IN BEVERIDGEAN MODEL

- Beveridgean business-cycle model (Michaillat & Saez 2022)
  1. Workers find customers through matching $\Rightarrow$ unemployment
  2. Utility from wealth $\Rightarrow$ nondegenerate aggregate demand

3. Price competition through directed search (Moen 1997)
4. Price rigidity from quadratic price-adjustment costs (Rotemberg 1982)

- In Beveridgean Phillips curve: $\pi = \pi^* \Leftrightarrow u = u^*$
- Other properties of the model:
  - Fluctuations in unemployment & inflation
  - Permanent zero-lower-bound episodes
  - Shocks to Beveridge curve $\rightsquigarrow$ shocks to Phillips curve
  - Kink to Phillips curve can be added

DESCRIPTION OF THE MODEL

UNEMPLOYED WORKERS & RECRUITERS

- People are organized in large guilds
- Each guild sell services to other guilds from their shop
  - Workers are full-time employees of customers
  - Employment relationships separate at rate $s > 0$
- Guild k has l_k members
  - y_{jk} members work for guild j
  - $y_k = \int_0^1 y_{jk}(t)\, dk$ members are employed in total
  - $U_k = l_k - y_k$ members are unemployed
- Guild j sends V_{jk} members from guild k to recruit other guild members
  - $V_k = \int_0^1 V_{jk}(t)\, dj$ recruiters are at shop k

MATCHING BETWEEN JOBSEEKERS & RECRUITERS

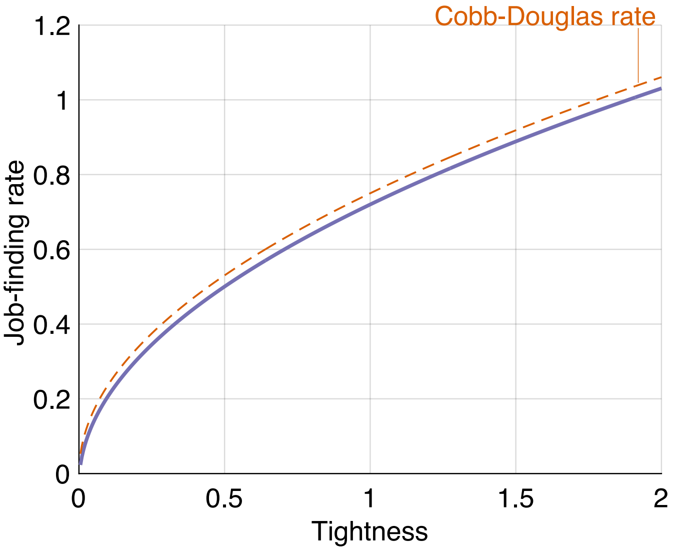
- Matching function determines flow of hires at shop k:

$$h_k = h(U_k, V_k) = \omega\sqrt{U_k \cdot V_k} - sU_k$$

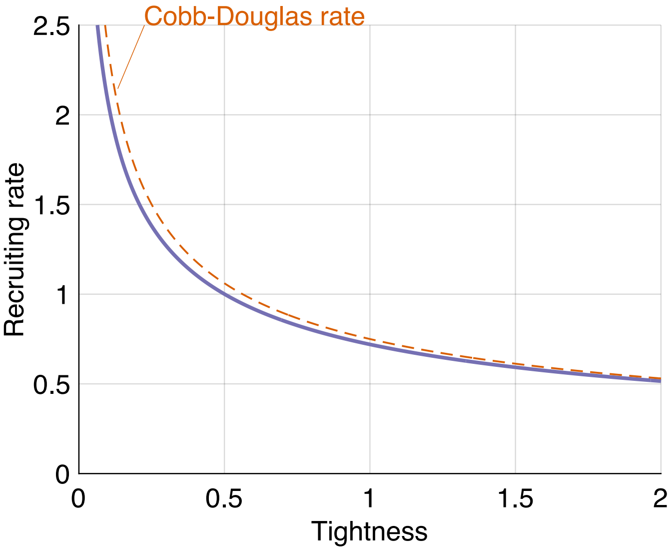
- Matching function has standard properties:
  - Constant returns to scale
  - $h = 0$ when $U = 0$
  - Increasing in V and U (as long as unemployment $< 50\%$)
  - Concave in V and U
- Market tightness $\theta_k = V_k/U_k$ determines matching rates
  - Job-finding rate: $f(\theta_k) = h_k/U_k = \omega\sqrt{\theta_k} - s$
  - Recruiting rate: $q(\theta_k) = h_k/V_k = \omega/\sqrt{\theta_k} - s/\theta_k$

US BEVERIDGE CURVE ≈ HYPERBOLA

MATCHING RATES BETWEEN WORKERS & CUSTOMERS



MATCHING RATES BETWEEN WORKERS & CUSTOMERS



BALANCED FLOWS & UNEMPLOYMENT RATE

- Assume that flows are balanced in guild k: $\dot{y}_k = 0$
- Law of motion of employment in guild k:

$$\dot{y}_k = f(\theta_k)U_k - s y_k = f(\theta_k)U_k - s\,(l_k - U_k)$$

- Local tightness and local unemployment rate are directly related:

$$u(\theta_k) \equiv \frac{U_k}{l_k} = \frac{s}{s + f(\theta_k)}$$

MODEL BEVERIDGE CURVE IS AN HYPERBOLA

- Balanced flows: $u_k = s/\left[s + f(\theta_k)\right]$
- Matching function: $f(\theta_k) = \omega\sqrt{\theta_k} - s$

$\leadsto$ $u_k = (s/\omega)/\sqrt{v_k/u_k}$

$\leadsto$ Beveridge curve is a rectangular hyperbola, just like in the US:

$$u_k \cdot v_k = (s/\omega)^2$$

BALANCED FLOWS & RECRUITER-PRODUCER RATIO

- Recruiters from guild k employed by guild j: V_{jk}
  - Their services do not deliver direct utility
- Producers from guild k employed by guild j: $c_{jk} = y_{jk} - V_{jk}$
  - Their services deliver direct utility
- Workers from guild k employed by guild j:

$$\dot{y}_{jk} = q(\theta_k)V_{jk} - sy_{jk} = q(\theta_k)V_{jk} - s\left(c_{jk} + V_{jk}\right)$$

- Assume that flows are balanced in all (j, k) cells: $\dot{y}_{jk} = 0$
- Local tightness determines the local recruiter-producer ratio:

$$\tau(\theta_k) \equiv \frac{V_{jk}}{c_{jk}} = \frac{s}{q(\theta_k) - s}$$

SOCIAL EFFICIENCY AT SHOP k

- Amount of services consumed:

$$c_k = y_k - V_k = l_k - U_k - V_k = l_k\,(1 - u_k - v_k)$$

- Maximizing c_k is equivalent to minimizing $u_k + v_k$
- Subject to the Beveridge curve $u_k \cdot v_k = (s/\omega)^2$
- From Michaillat & Saez (2024), the solution to the maximization is:

$$u_k^* = \sqrt{u_k \cdot v_k} = s/\omega, \qquad \theta_k^* = 1$$

COMPETITION THROUGH DIRECTED SEARCH

- Guild k charges price p_k per unit time
- Expenditure by guild j on workers k is

$$p_k y_{jk} = p_k \left(c_{jk} + V_{jk}\right) = p_k \left[1 + \tau(\theta_k)\right] c_{jk}$$

- Workers are perfectly substitutable
  - Only $c_j = \int_0^1 c_{jk}(t)\, dk$ enters the utility function
- $p_k \left[1 + \tau(\theta_k)\right]$ must be the same across guilds
- There is a price level p so $p_k \left[1 + \tau(\theta_k)\right] = p \left[1 + \tau(\theta)\right]$ for all k

EFFECT OF LOCAL PRICE ON LOCAL TIGHTNESS

- Price chosen by guild k determines the tightness it faces:

$$\theta_k = \tau^{-1}\left(\frac{p}{p_k}\left[1 + \tau(\theta)\right] - 1\right)$$

- The function τ^{-1} is increasing, so θ_k is decreasing in p_k

⇝ High price leads to low tightness, high unemployment

⇝ Low price leads to high tightness, low unemployment

EFFICIENCY WITHOUT PRICE RIGIDITY (MOEN 1997)

- Guild chooses price to maximize income subject to demand curve
- Subject to demand $\theta_k(p_k)$, guild chooses p_k to maximize:

$$p_k y_k = p\left[1+\tau(\theta)\right]\frac{y_k}{1+\tau(\theta_k)} = p\left[1+\tau(\theta)\right] l_k \frac{1-u(\theta_k)}{1+\tau(\theta_k)}$$

- $\tau(\theta), u(\theta), v(\theta)$ are linked by

$$\frac{1-u(\theta_k)}{1+\tau(\theta_k)} = 1-u(\theta_k)-v(\theta_k)$$

- Guild sets tightness θ_k to minimize $u(\theta_k)+v(\theta_k)$

$\Leftrightarrow$ Sets unemployment rate u_k to minimize $u_k + v(u_k)$

$\Leftrightarrow$ Unemployment rate u_k is socially efficient

PRICE RIGIDITY

- Unexpected price/wage changes upset customers/workers
  - Shiller (1996): higher-than-normal price inflation upsets customers, who feel unfairly treated when they go to the store
  - Bewley (1999): lower-than-normal wages damage workers' morale, who feel unfairly treated
- Inflation chosen by guild k: $\pi_k = \dot{p}_k / p_k$
- Flow disutility when inflation deviates from norm (Rotemberg 1982):

$$\frac{\kappa}{2}\left(\pi_k - \pi^*\right)^2$$

- $\kappa > 0$: price-adjustment cost

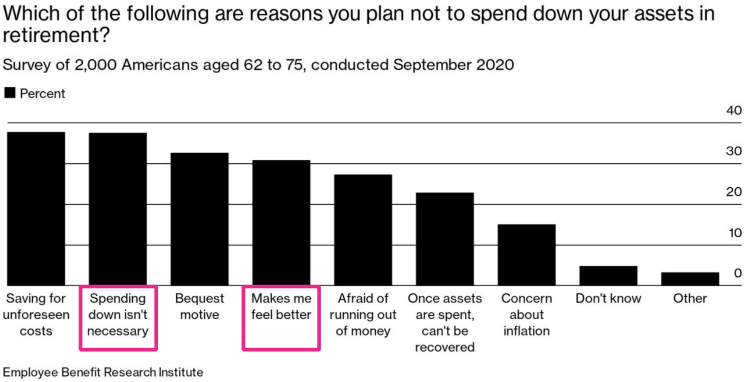
GUILD'S PREFERENCES

- Guild j maximizes utility

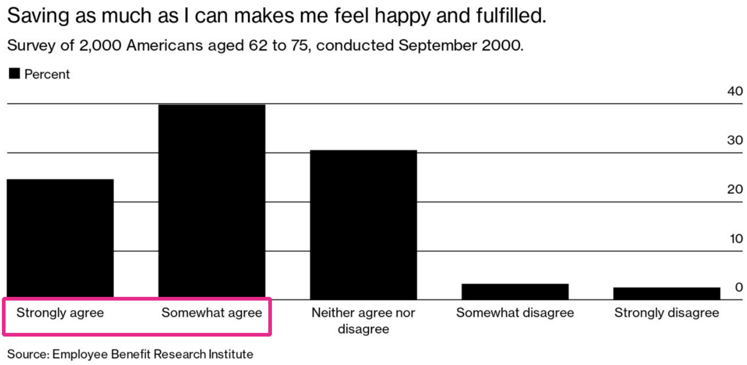
$$\int_0^\infty e^{-\delta t}\left\{\ln\big(c_j(t)\big)+\sigma\left[\frac{b_j(t)}{p(t)}-\frac{b(t)}{p(t)}\right]-\frac{\kappa}{2}\left[\pi_j(t)-\pi^*\right]^2\right\}dt$$

- $\delta > 0$: time discount rate
- $c_j(t) = \int_0^1 c_{jk}(t)\,dk$: total consumption of services
- $b_j(t)$: saving in government bonds
- $b(t) = \int_0^1 b_j(t)\,dj$: aggregate wealth
- $\sigma > 0$: status concerns

“THE VIRTUE OF THE CAKE WAS THAT IT WAS NEVER TO BE CONSUMED, NEITHER BY YOU NOR BY YOUR CHILDREN AFTER YOU” (KEYNES 1919)



“THE VIRTUE OF THE CAKE WAS THAT IT WAS NEVER TO BE CONSUMED, NEITHER BY YOU NOR BY YOUR CHILDREN AFTER YOU” (KEYNES 1919)



GUILD'S BUDGET CONSTRAINT

- Law of motion of government bond holdings for guild j:

$$\dot{b}_j = ib_j - \int_0^1 p_k y_{jk}\, dk + p_j y_j$$

- With matching and directed search, expenditure becomes:

$$\begin{aligned}\int_0^1 p_k y_{jk}\, dk &= \int_0^1 p_k \left[1 + \tau(\theta_k)\right] c_{jk}\, dk \\ &= p\left[1 + \tau(\theta)\right] \int_0^1 c_{jk}\, dk \\ &= p[1 + \tau(\theta)]c_j\end{aligned}$$

- With matching and directed search, income becomes:

$$p_j y_j = p_j[1 - u(\theta_j(p_j))]l_j$$

SOLUTION OF THE MODEL

SOLVING GUILD'S PROBLEM BY HAMILTONIAN

- Hamiltonian of guild j's maximization is

$$\begin{aligned}\mathcal{H}_j = \; & \ln\left(c_j\right) + \sigma\left[\frac{b_j}{p} - \frac{b}{p}\right] - \frac{\kappa}{2}[\pi_j - \pi^*]^2 \\ & + \mathcal{A}_j\left[ib_j - p[1+\tau]c_j + p_j[1 - u(\theta_j(p_j))]l_j\right] \\ & + \mathcal{B}_j \pi_j p_j.\end{aligned}$$

- Control variables: c_j, π_j
- State variables: b_j, p_j
- Costate variables: $\mathcal{A}_j, \mathcal{B}_j$
- Symmetric solution of the model: all guilds behave identically

AGGREGATE SUPPLY: PHILLIPS EQUATION

- From optimal pricing by guilds:

$$\dot{\pi} = \delta(\pi - \pi^*) - \frac{1}{\kappa}\left[1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}\right]$$

- $1 - \frac{u \cdot (1-u-v)}{v \cdot (1-2u)}$: inefficiency of the economy
  - Zero $\Leftrightarrow u = v \Leftrightarrow \theta = 1 \Leftrightarrow$ efficiency
  - Positive $\Leftrightarrow v > u \Leftrightarrow \theta > 1 \Leftrightarrow$ inefficiently tight
  - Negative $\Leftrightarrow u > v \Leftrightarrow \theta < 1 \Leftrightarrow$ inefficiently slack
- In steady state ($\dot{\pi} = 0$), Phillips curve:

$$\kappa\delta(\pi - \pi^*) = 1 - \frac{u}{v(u)} \cdot \frac{1 - u - v(u)}{1 - 2u}$$

AGGREGATE DEMAND: EULER EQUATION

- From optimal consumption and saving by guilds:

$$\frac{\dot{u}}{1-u} = \delta - [i(\pi) - \pi + \sigma(1-u)l]$$

- $i(\pi) - \pi$ = real interest rate = financial return on saving
- $\sigma(1-u)l$ = MRS between wealth & consumption = hedonic return on saving
- In steady state ($\dot{u} = 0$), Euler curve:

$$\pi = i(\pi) - \delta + \sigma \cdot (1-u) \cdot l$$

DIVINE COINCIDENCE WITH BEVERIDGEAN PHILLIPS CURVE

- Beveridgean Phillips curve:

$$\kappa \cdot \delta \cdot (\pi - \pi^*) = 1 - \frac{u}{v} \cdot \frac{1 - u - v}{1 - 2u}$$

- $\pi = \pi^* \Leftrightarrow u = v \Leftrightarrow \theta = 1 \Leftrightarrow u = u^*$
- $(u^*, \pi^*) \in$ Beveridgean Phillips curve

⇝ Inflation is on target whenever unemployment is efficient

⇝ Price-stability and full-employment mandates are consistent

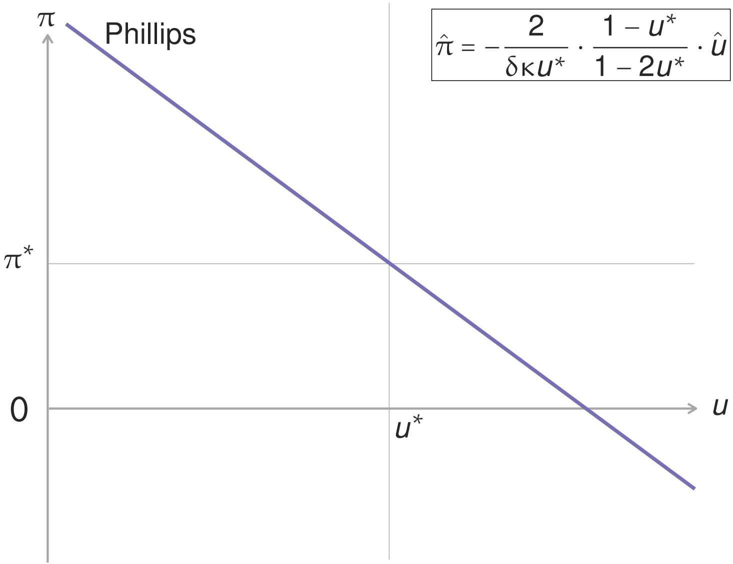
⇝ Divine coincidence holds

MONETARY POLICY SATISFYING THE DUAL MANDATE

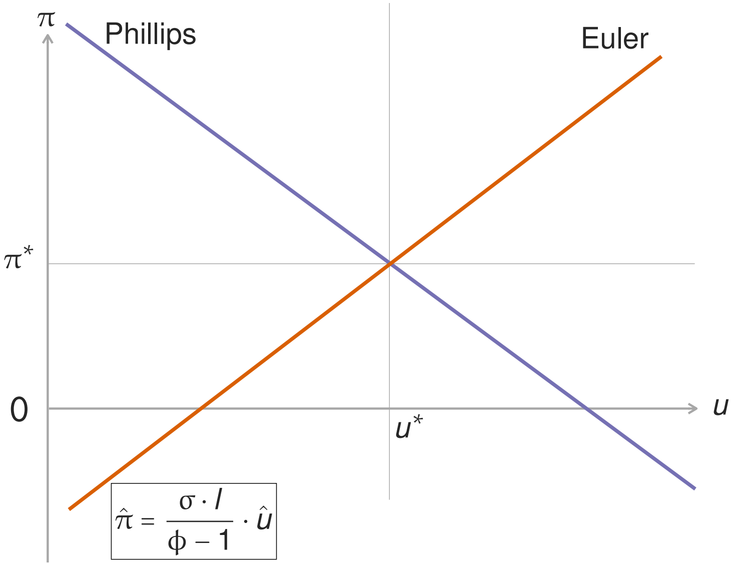
- Nominal interest rate i^* ensures:
  - Inflation is on target: $\pi = \pi^*$
  - Unemployment is efficient: $u = u^*$
- From Euler curve: $i^* = \pi^* + \delta - \sigma \cdot (1 - u^*) \cdot l$
- policy can take different forms:
  - Interest-rate peg: $i(\pi) = i^*$
  - Taylor rule with $\phi > 0$: $i(\pi) = i^* + \phi \cdot (\pi - \pi^*)$
- Dual-mandate policy also maximizes social welfare

DYNAMICS OF THE MODEL

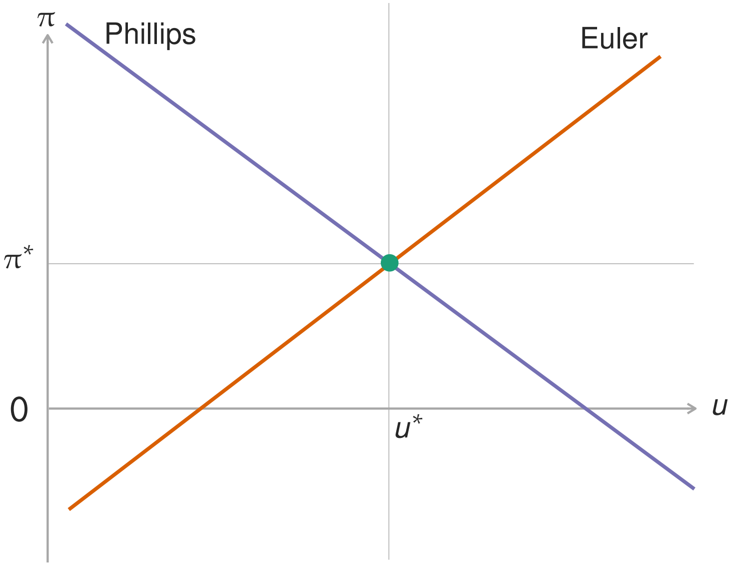
LINEARIZED PHILLIPS CURVE



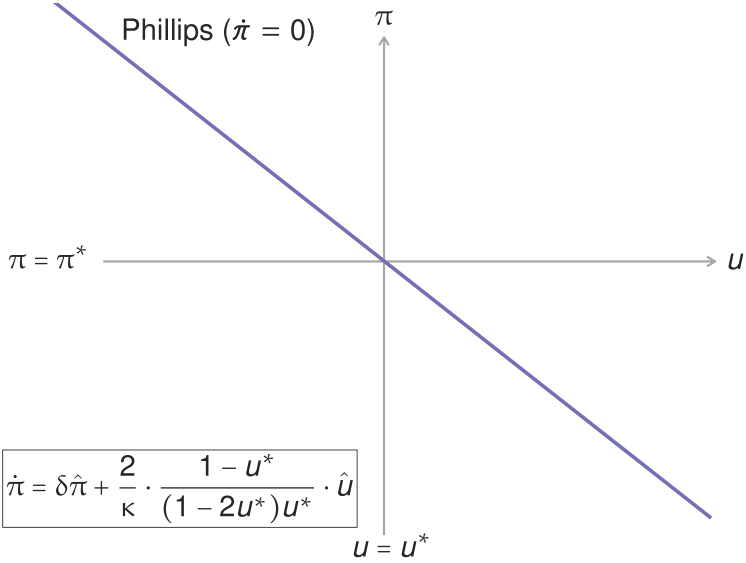
LINEARIZED EULER CURVE



DIVINE COINCIDENCE

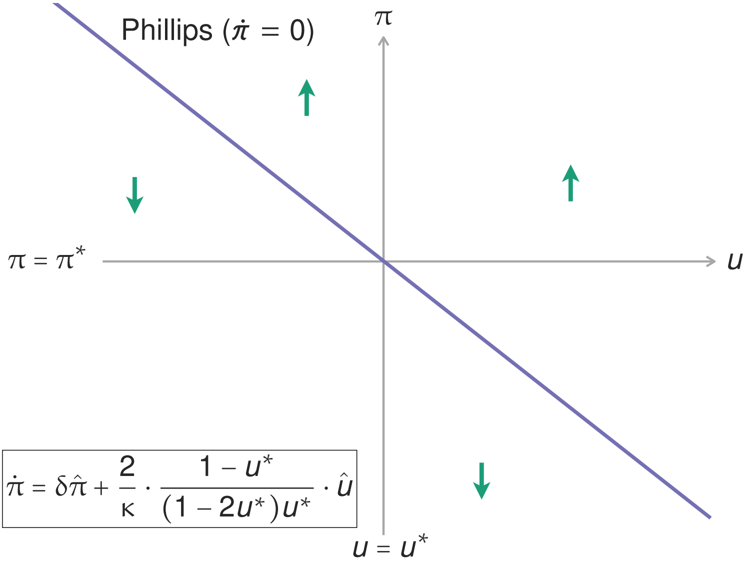


PHASE DIAGRAM



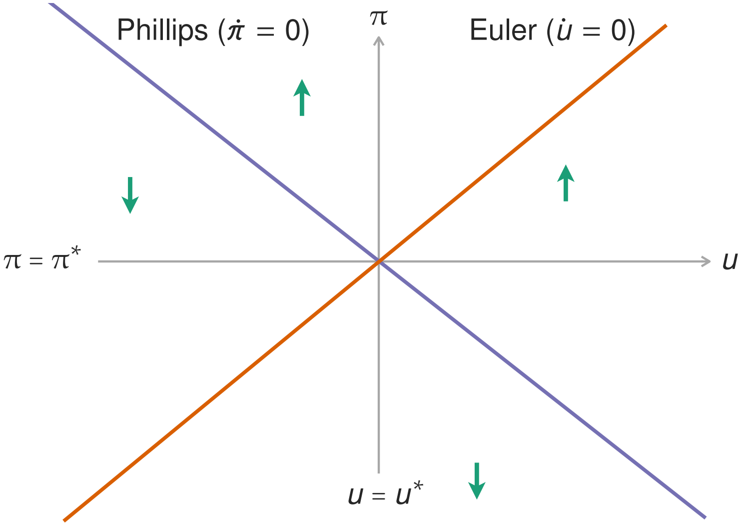
$$\boxed{\dot{\pi} = \delta\hat{\pi} + \frac{2}{\kappa} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}}$$

PHASE DIAGRAM



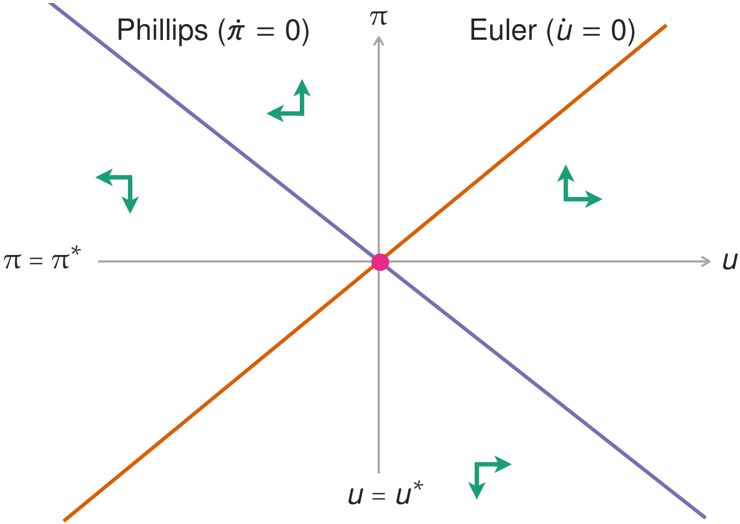
$$\boxed{\dot{\pi} = \delta\hat{\pi} + \frac{2}{\kappa} \cdot \frac{1 - u^*}{(1 - 2u^*)u^*} \cdot \hat{u}}$$

PHASE DIAGRAM: TAYLOR RULE



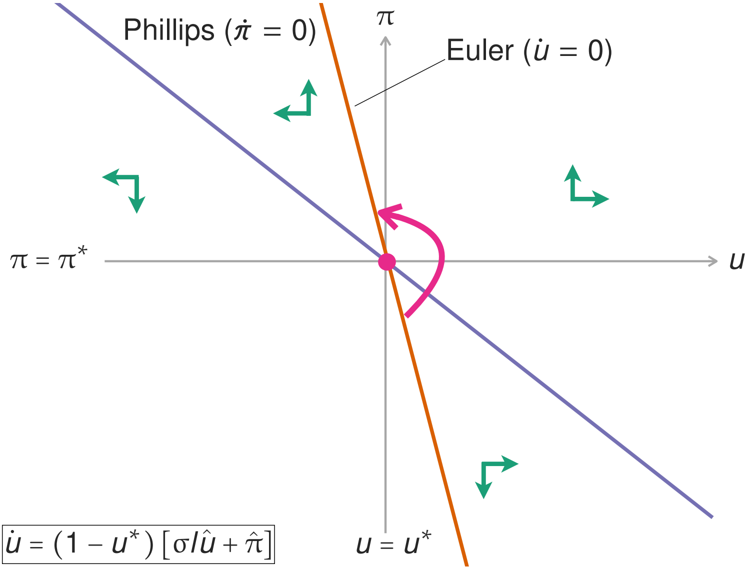
$$\boxed{\dot{u} = (1 - u^*)\left[\sigma/\hat{u} - (\phi - 1)\hat{\pi}\right]}$$

PHASE DIAGRAM: TAYLOR RULE



$$\boxed{\dot{u} = (1 - u^*)\left[\sigma/\hat{u} - (\phi - 1)\hat{\pi}\right]}$$

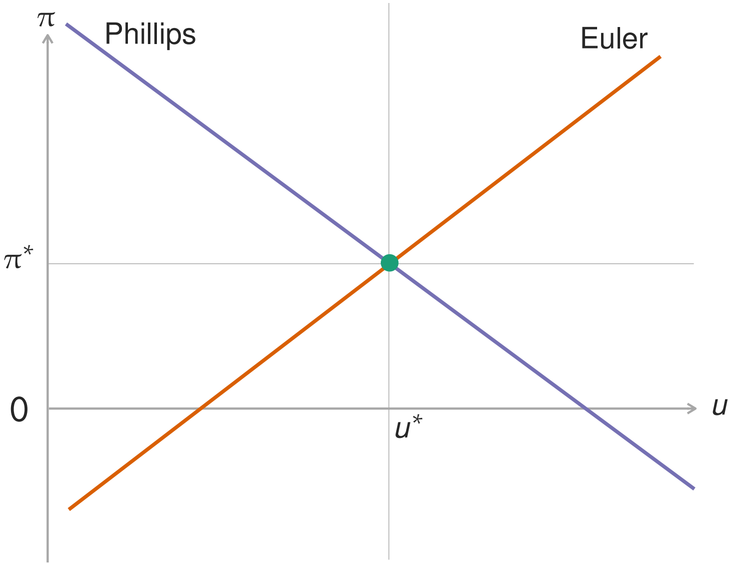
PHASE DIAGRAM: ZLB



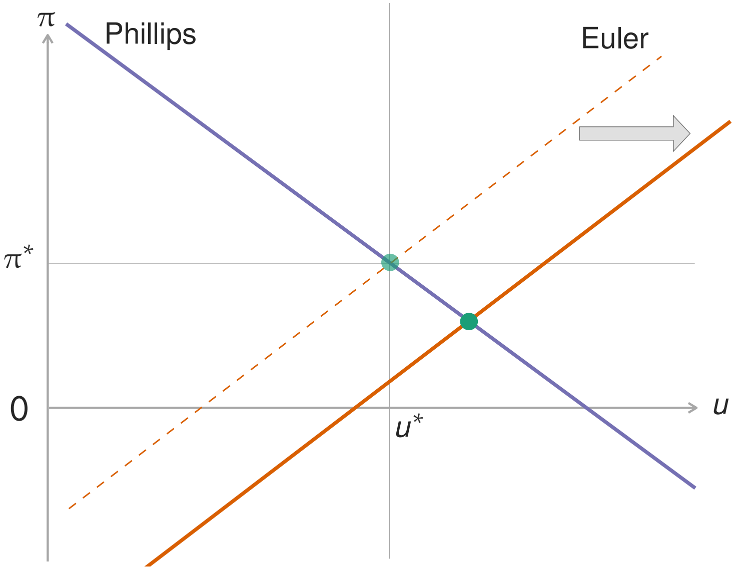
$$\boxed{\dot{u} = (1 - u^*)\left[\sigma/\hat{u} + \hat{\pi}\right]}$$

RESPONSE TO DEMAND AND SUPPLY SHOCKS

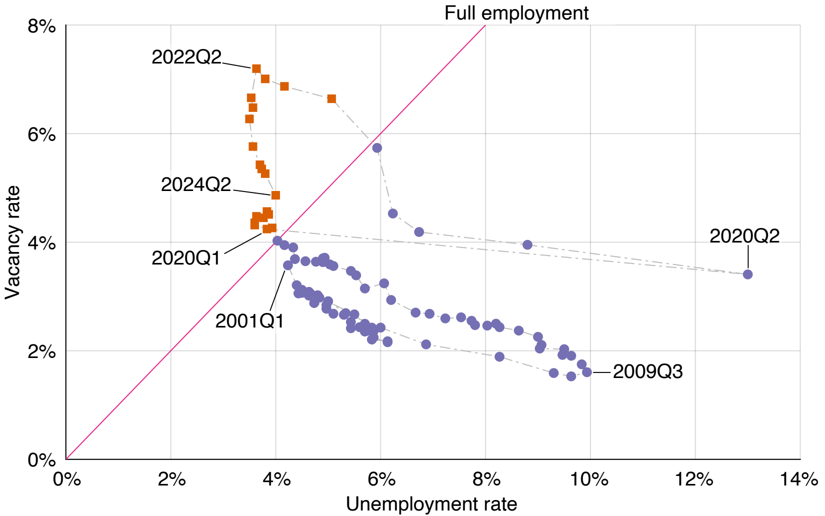
NEGATIVE DEMAND OR MONETARY SHOCK



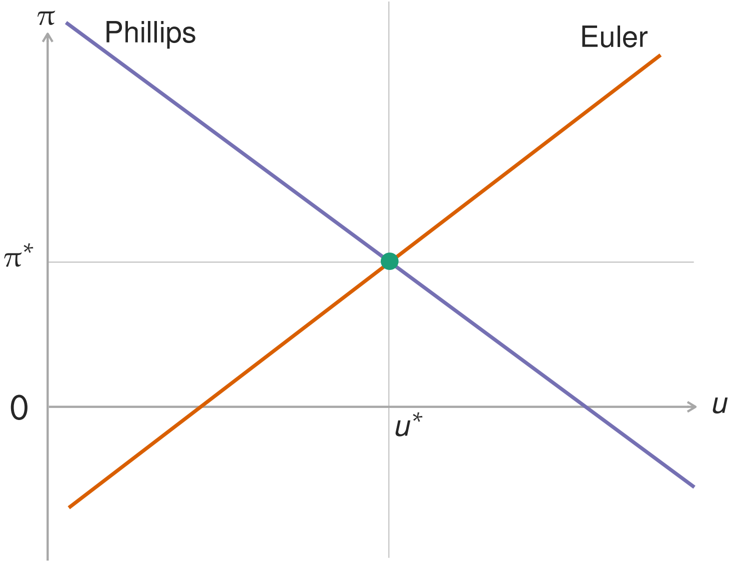
NEGATIVE DEMAND OR MONETARY SHOCK



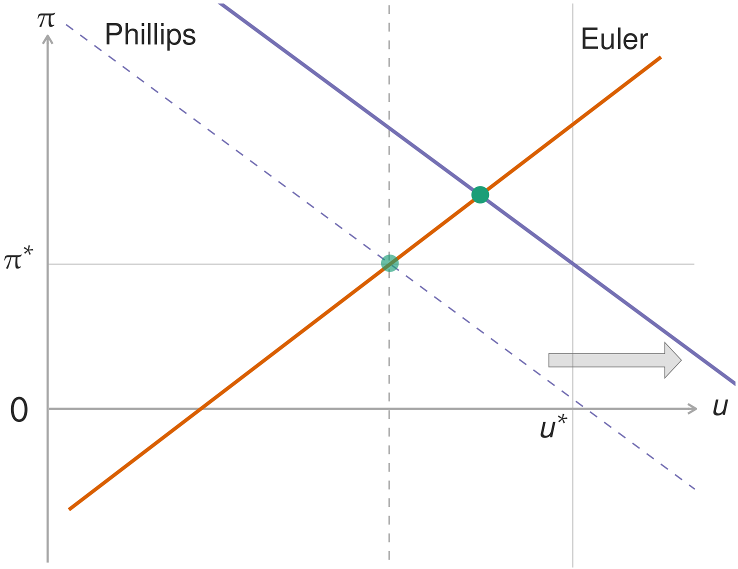
PANDEMIC SHIFT OF THE BEVERIDGE CURVE



PANDEMIC BEVERIDGE SHIFT: HIGH u, HIGH π

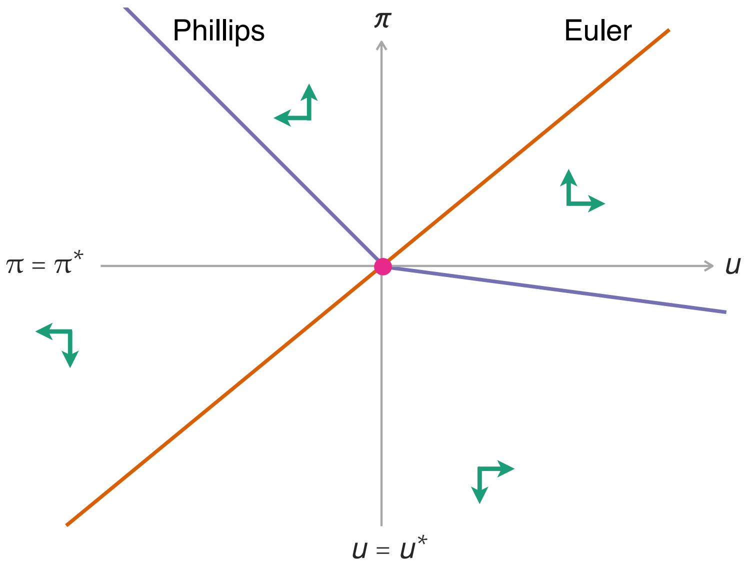


PANDEMIC BEVERIDGE SHIFT: HIGH u, HIGH π

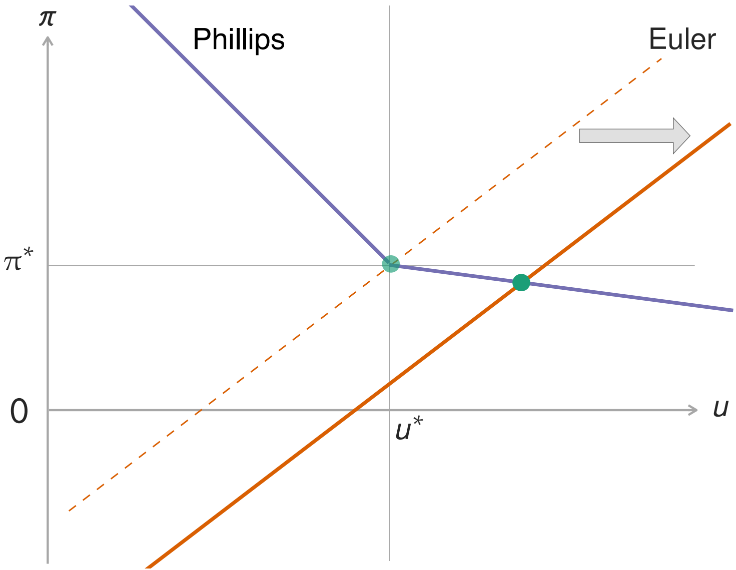


KINK IN THE PHILLIPS CURVE

DOWNWARD WAGE RIGIDITY > UPWARD PRICE RIGIDITY



NEGATIVE DEMAND SHOCK: UNEMPLOYMENT GAP ↑



NEGATIVE SUPPLY SHOCK: INFLATION ↑